Stress in Static Sandpiles

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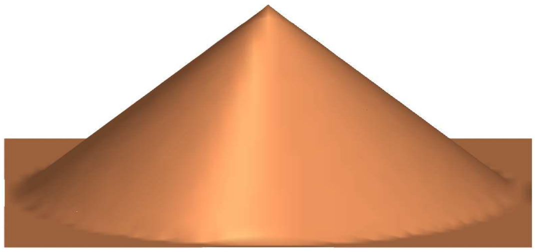
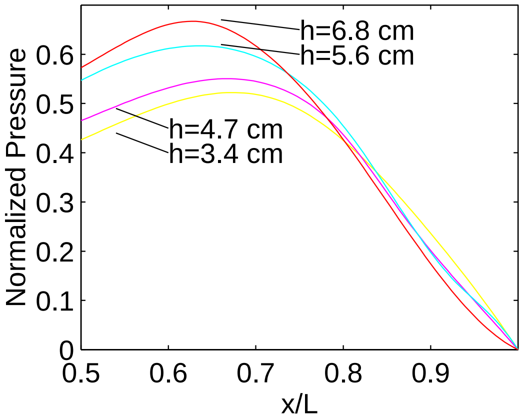
Questions

- What is the pressure distribution below sandpiles ?
- How can I get information from the inside ?
- How can I calculate the pressure distribution ?
- What can be stated about continuum theories now ?

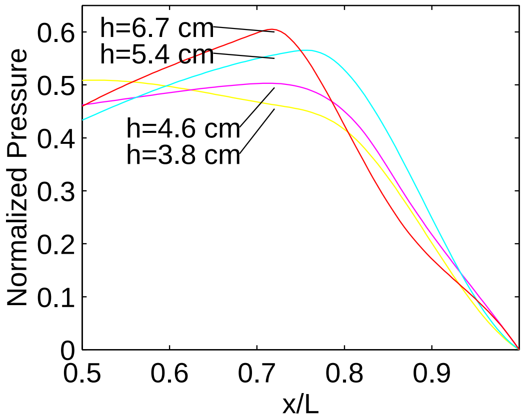
Sandpiles



Ground pressure



rape seed



sea sand

Pressure minimum in the center of the pile $\longrightarrow$ DIP

DIP depends on:

- the shape of the sandpile (wedge or pile)
- the preparation and history (layered or wedge sequence)
- the particle geometry (roughness, friction, etc.)

Continuum Theories

$$\partial_r \sigma_{rr} + \partial_z \sigma_{rz} = 0$$
$$\partial_r \sigma_{rz} + \partial_z \sigma_{zz} = \rho g$$

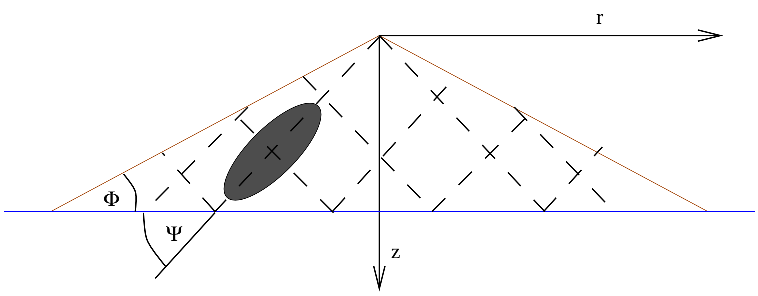
2 equations — 3 unknowns

HOOKE's law does not hold in general for granular media.

$\rightarrow$ Search for relations between the elements of the stress tensor

FPA Fixed Principal Axis

Assumption: The principal axis of the stress tensor has a fixed inclination



J.P. Wittmer et al. cond-mat9607097

2D MD-Simulation

Particle modeling

- "soft" particles
- polygons instead of spheres/circles
- disordered shapes and various sizes of the particles

Force laws

- contact force
- static friction
- dissipation

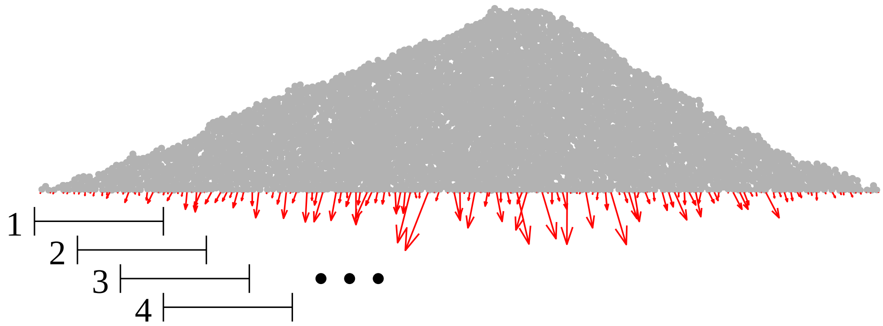
Differential-Equation-Solver

- higher order: less computation time
- Gear-Predictor-Corrector: high stability

Numerical Properties

- runtime $\propto$ particle number
- runtime constant for arbitrary number of corners

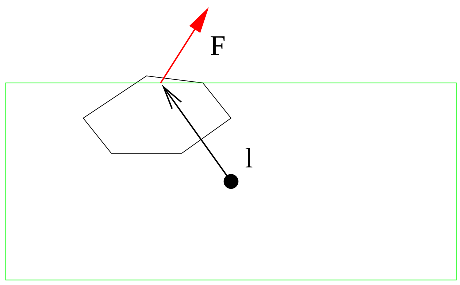
Calculation of the pressure



$$p_i = \frac{1}{l_i} \sum_{k \in i} F_k^{\perp}$$

Calculation of the stress tensor

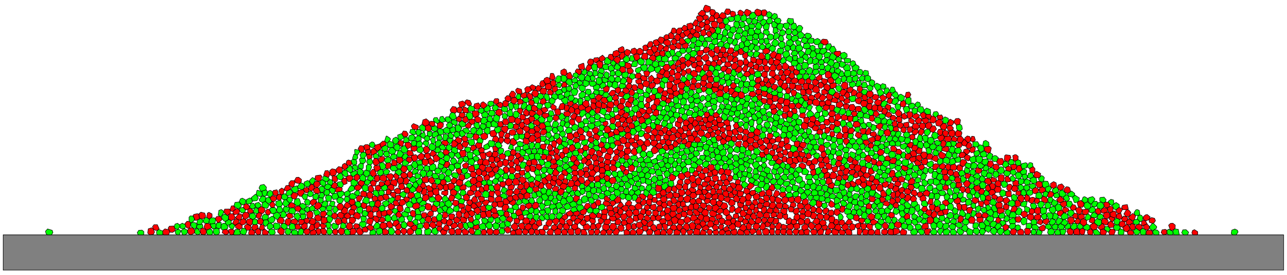
$$\sigma_{ij} = \frac{1}{2V} \sum (l_i F_j + l_j F_i)$$



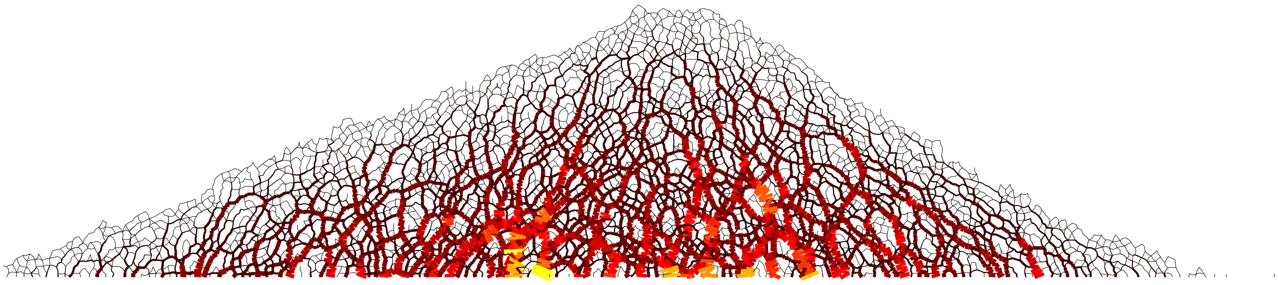
Heap 1

- Wedge Sequence
- 3336 particles
- 7 corners
- $r_a = 0.0095 \pm 15\%$, $r_b = 0.0095 \pm 15\%$

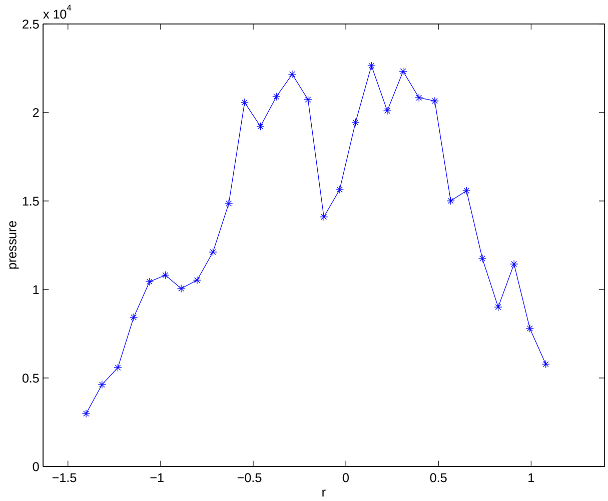
History



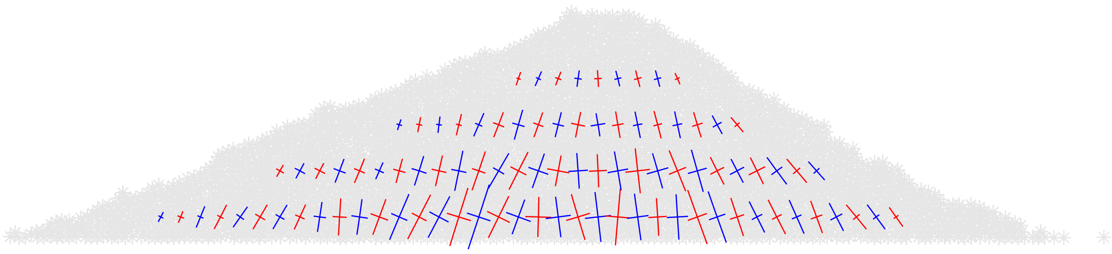
Force network



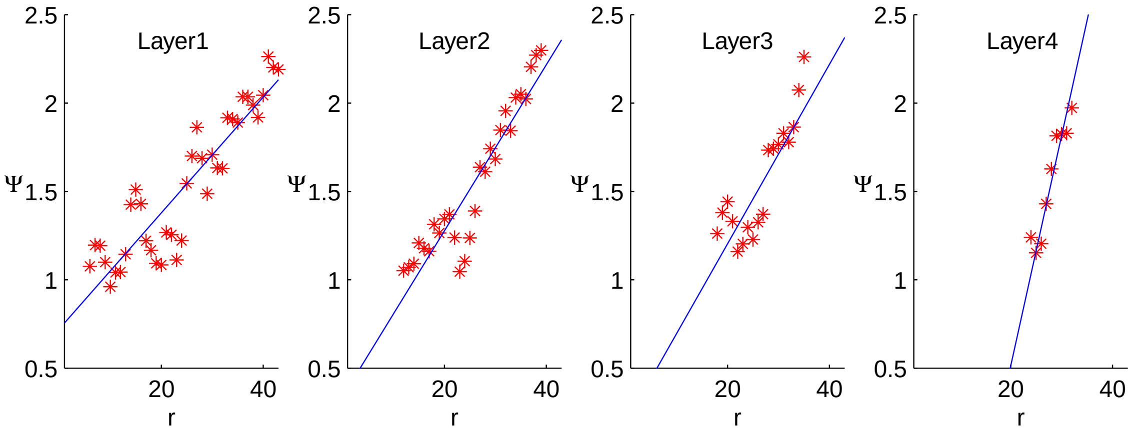
Ground Pressure



Stress tensors



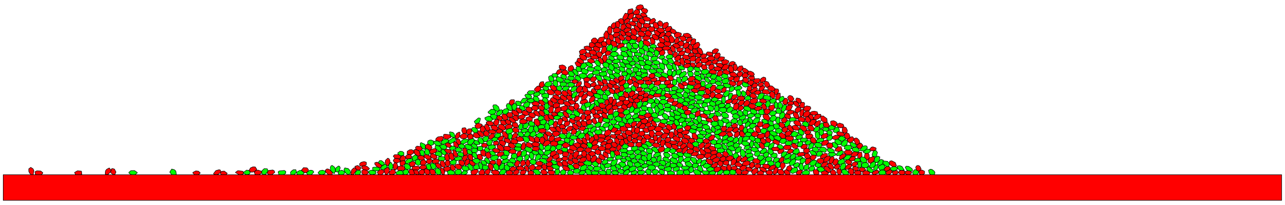
Angle of major Principal Axis



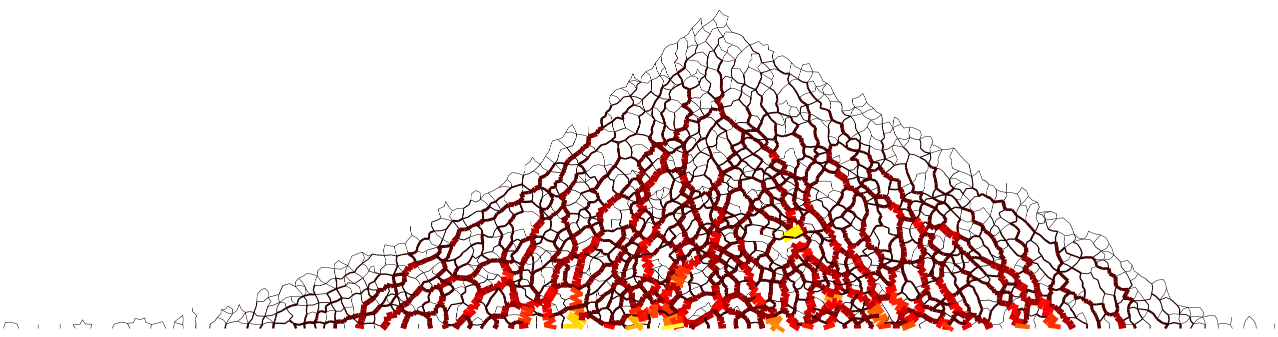
Heap 2

- Wedge Sequence
- 1502 particles
- 8 ± 1 corners
- $r_a = 0.0095 \pm 15\%$, $r_b = 0.0145 \pm 15\%$

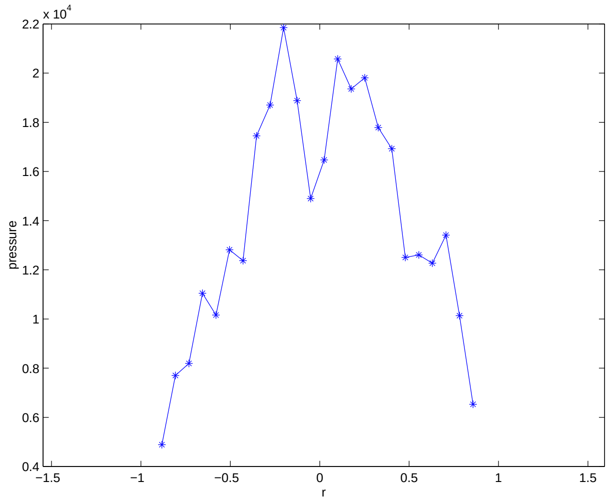
History



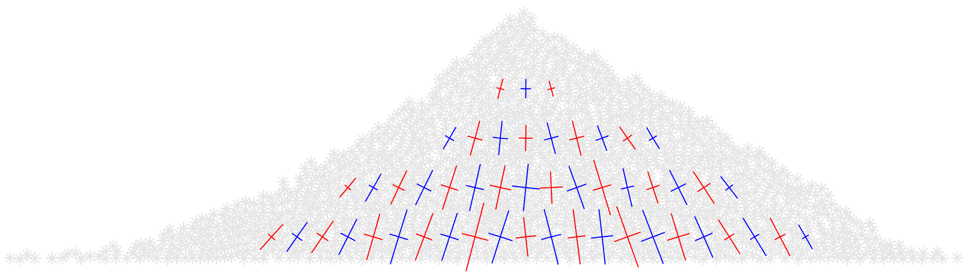
Force network



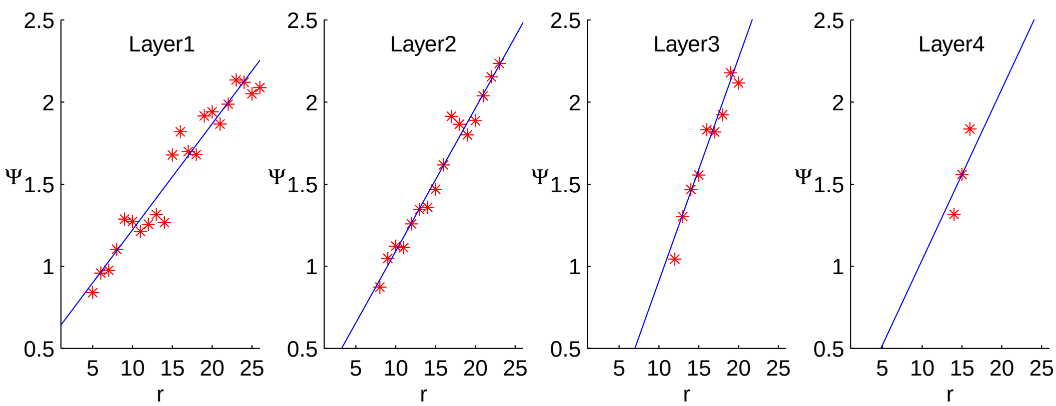
Ground Pressure



Stress tensors



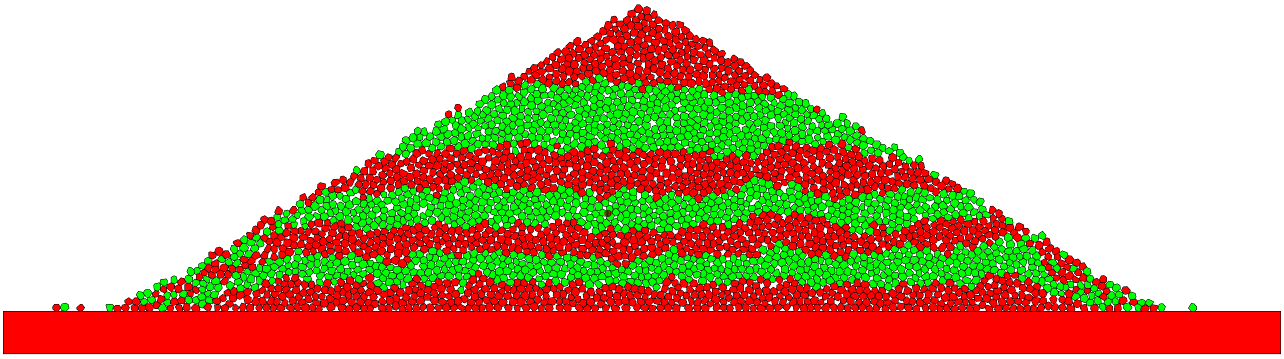
Angle of major Principal Axis



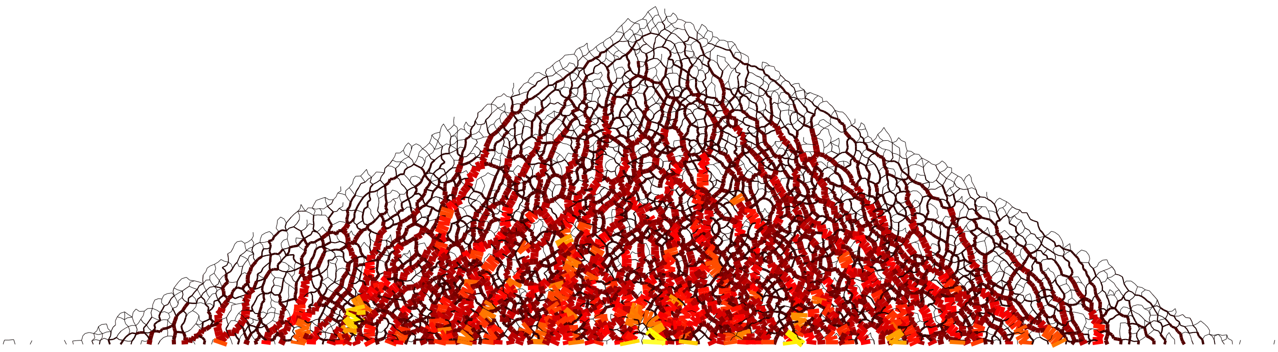
Heap 3

- Layered Sequence
- 3249 particles
- 7 corners
- $r_a = 0.0095 \pm 15\%$, $r_b = 0.0095 \pm 15\%$

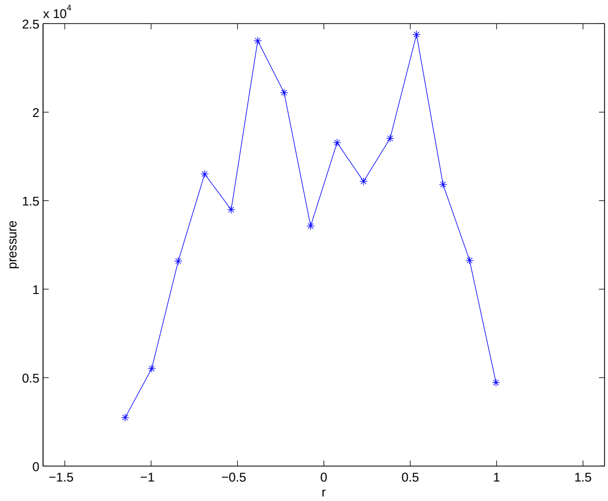
History



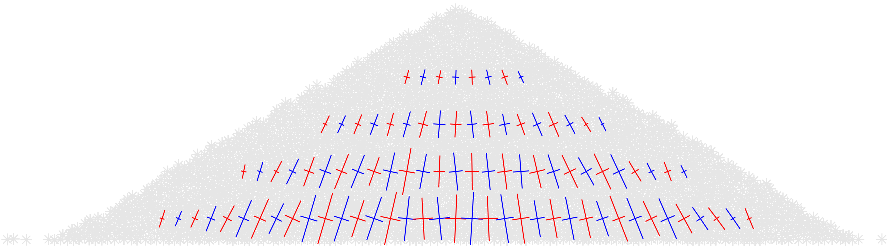
Force network



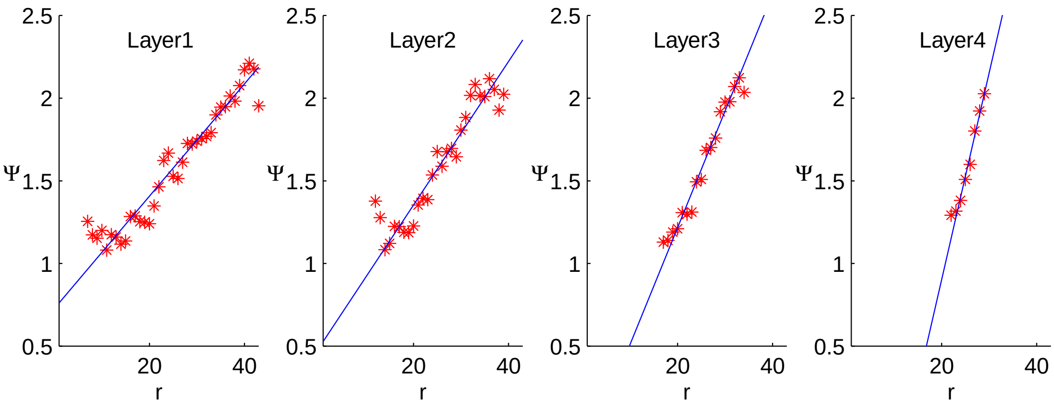
Ground Pressure



Stress tensors



Angle of major Principal Axis



Summary

1. Inspite of a rigid ground there *is* a DIP.

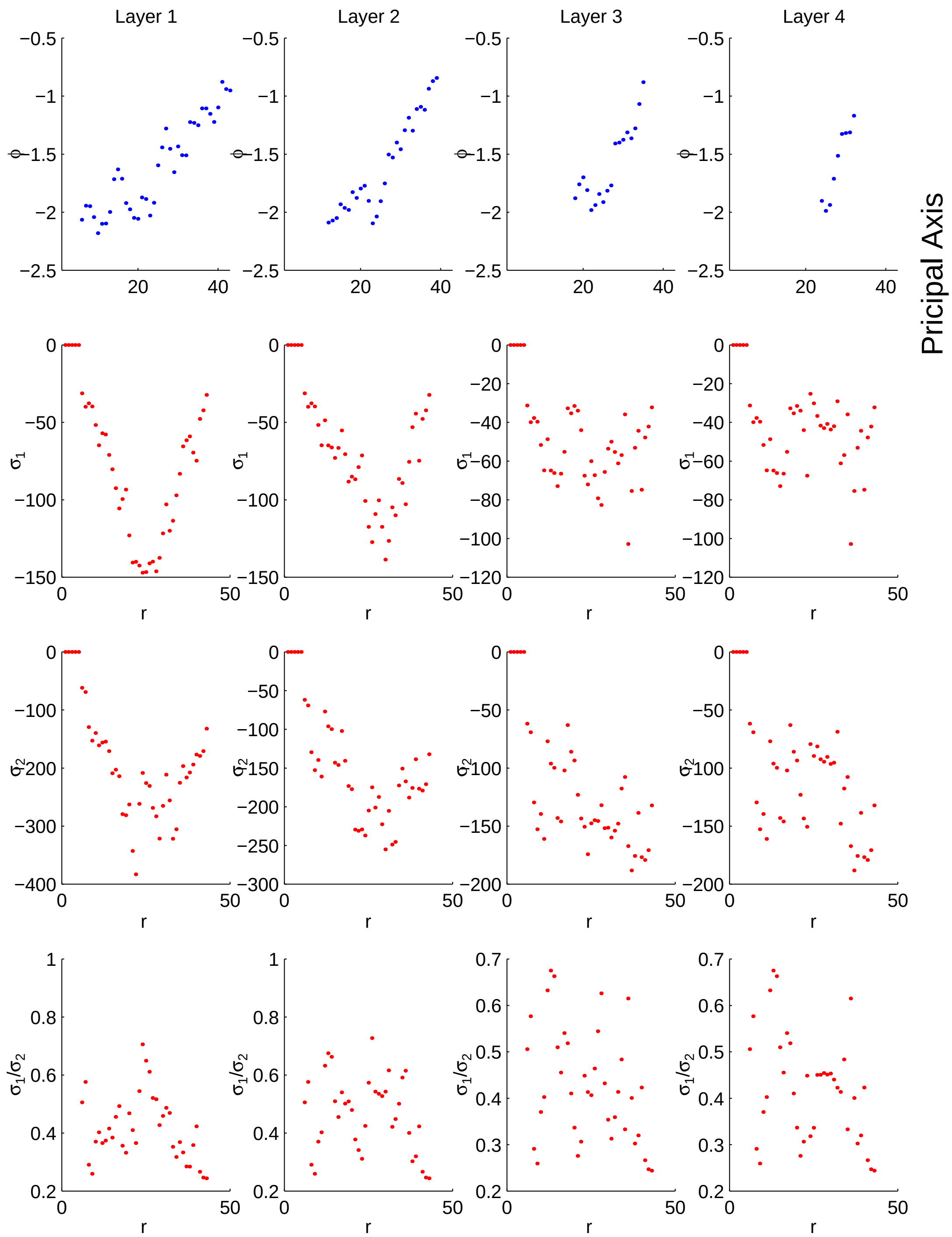
   see H.-G. Matuttis, Simulation of the pressure distribution under a two-dimensional heap of polygonal particles, **Granular Matter**

2. Calculated stress tensors contradict the FPA-Assumption.

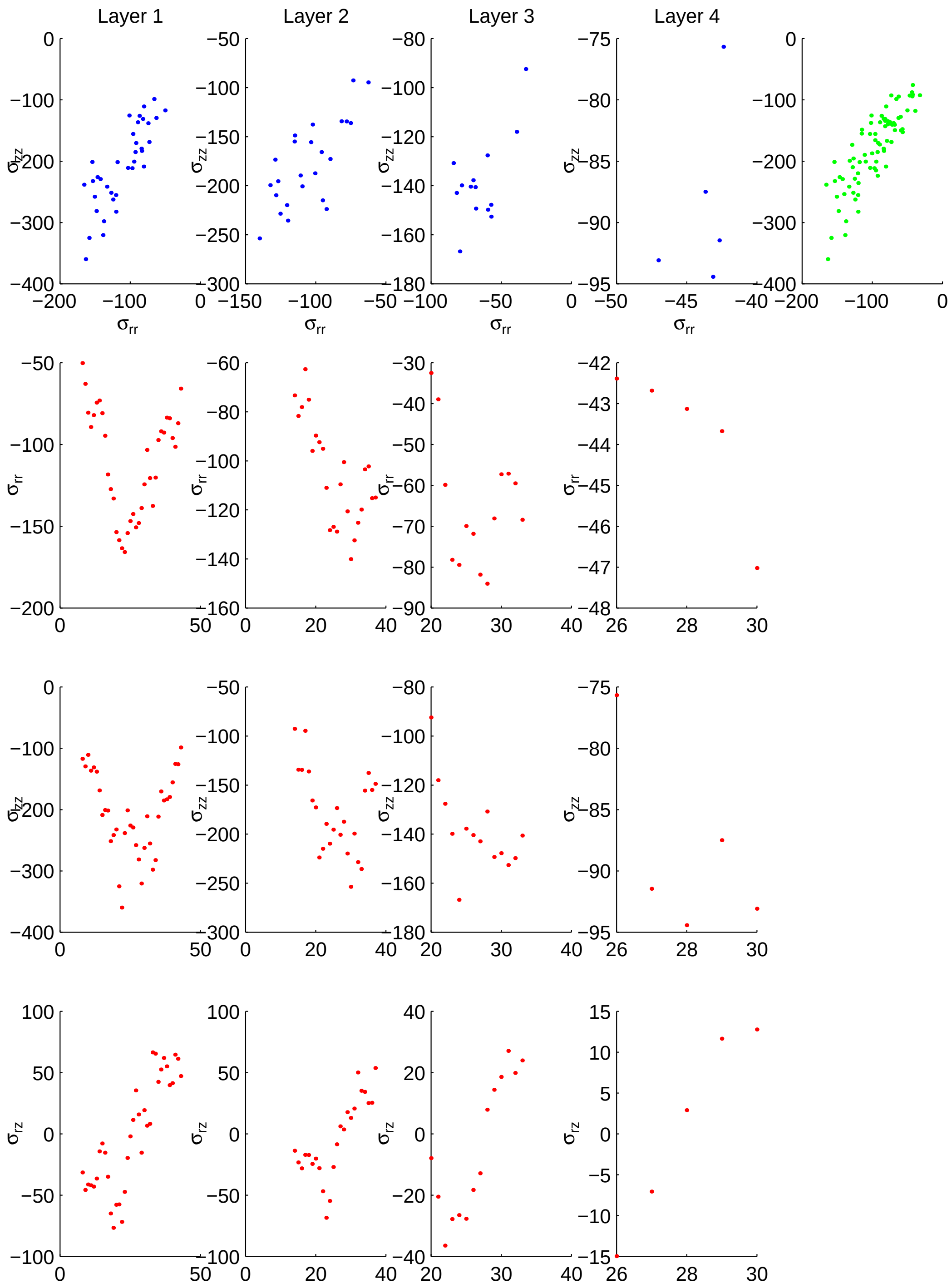
3. Principal stresses components are linearly related to each other.

4. Inside the sandpile the inclination of the principal axis is changing linearly with the radius r.

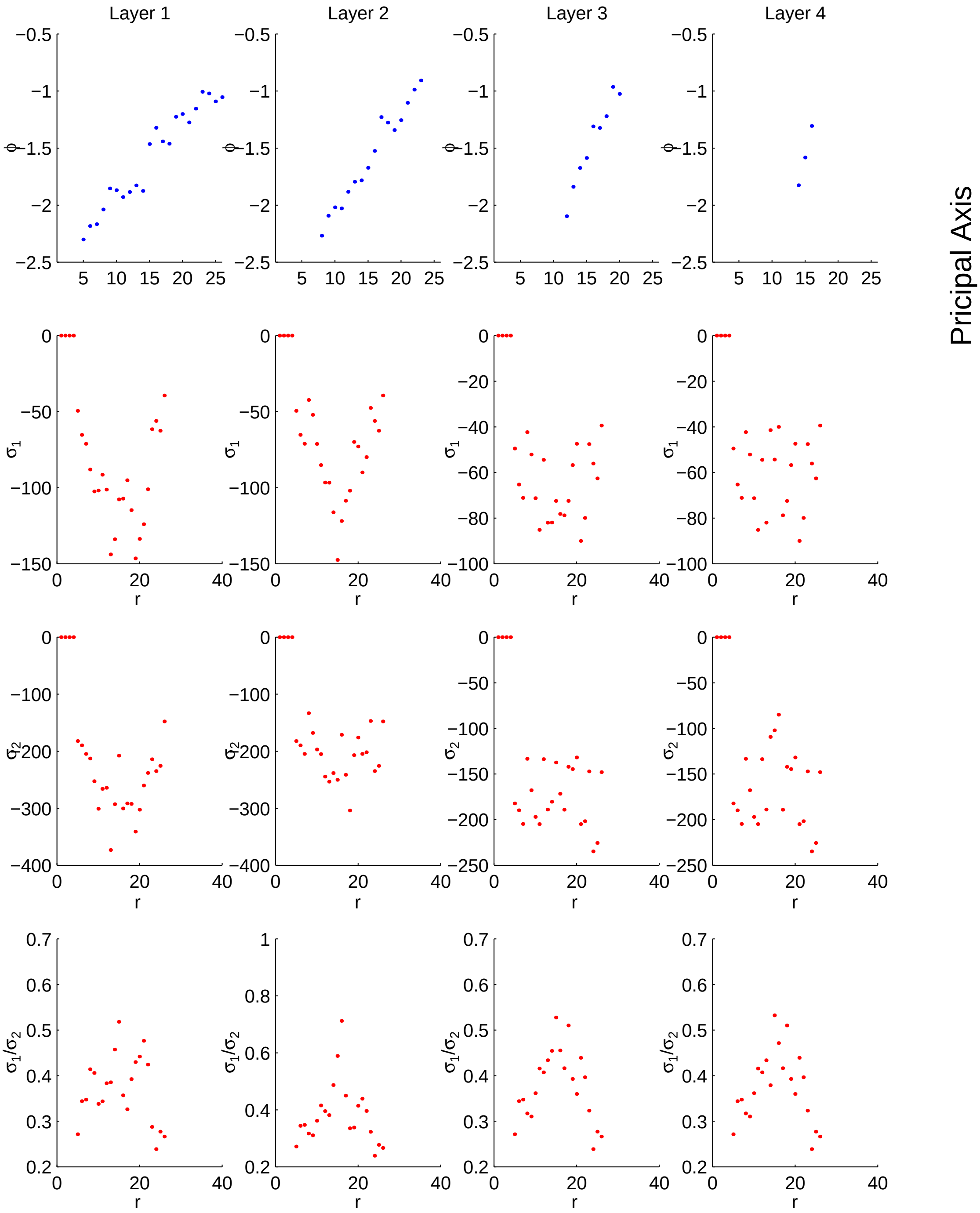
Principal Axis, Heap 1



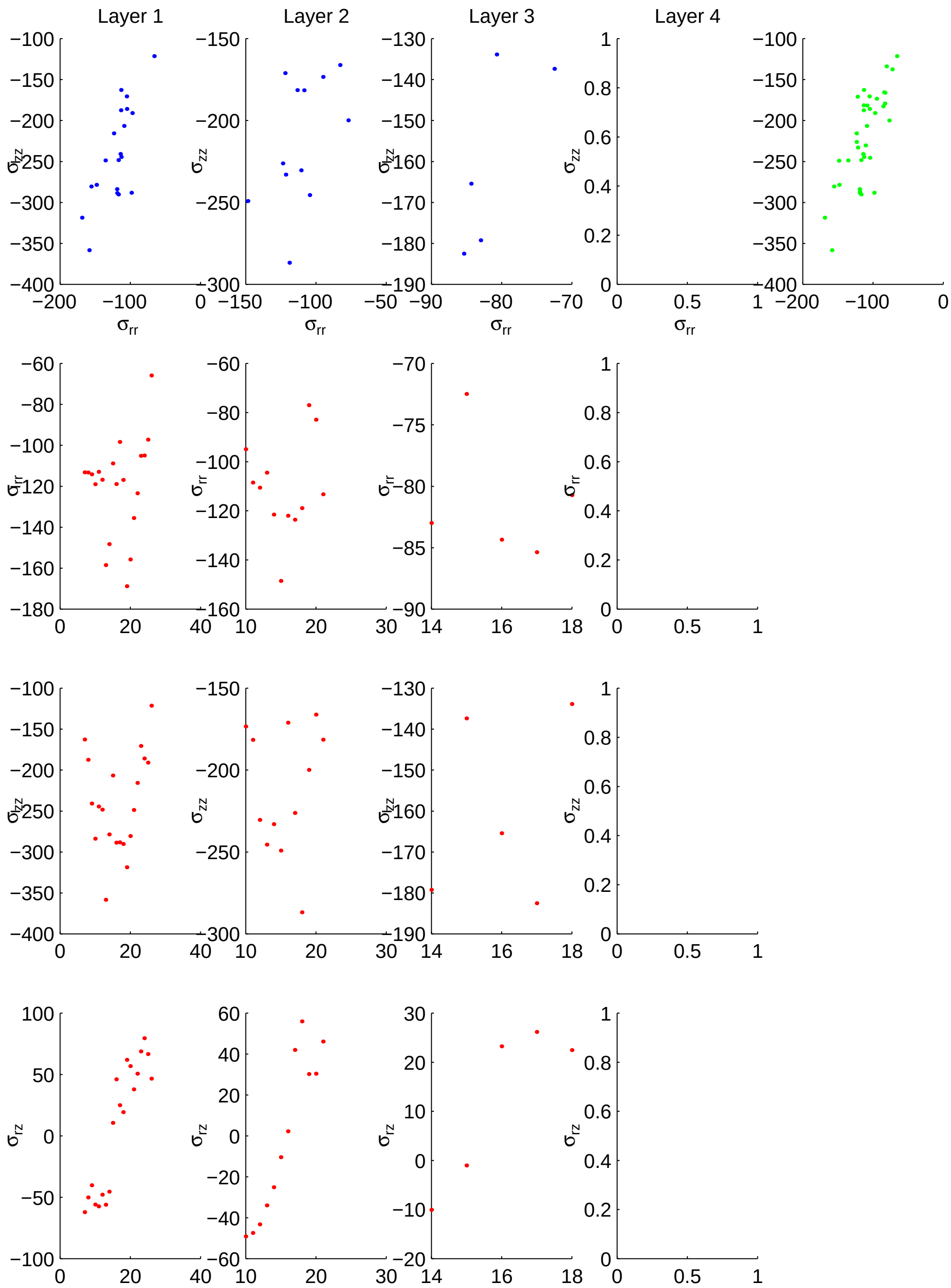
Stress, Heap 1



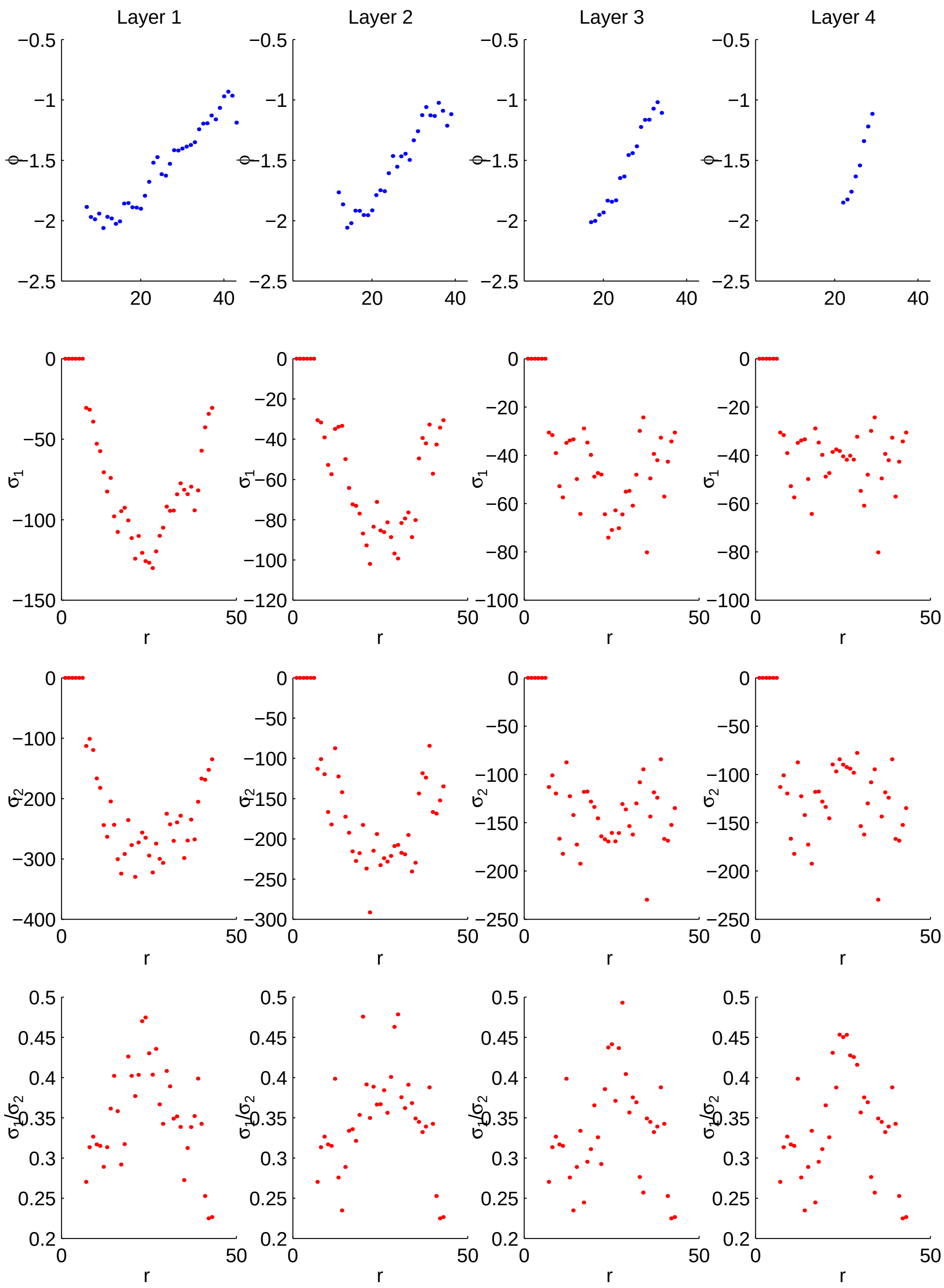
Principal Axis, Heap 2



Stress, Heap 2



Principal Axis, Heap 3



Stress, Heap 3